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Snap-through actuation of thick-wall electroactive balloons

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ABSTRACT

Solution to the problem of a spherical balloon made out of an electroactive polymer which is subjected to coupled mechanical and electrical excitations is determined. It is found that for certain material behaviors instabilities that correspond to abrupt changes in the balloon size can be triggered. This can be exploited to electrically control different actuation cycles as well as to use the balloon as a micro-pump.

Electroactive polymers (EAP) are materials that change their size and shape in response to electrostatic excitation. Roughly speaking, the electrostatically induced Maxwell stress results in the deformation of the material. Various EAP based actuators have been considered in the past [1–7]. A major limitation of these materials originates in the need for relatively large electric fields [8]. However, the usage of instability phenomena [9–13] may reduce the intensity of the required electric field. Accordingly, we analyze the response of EAP balloons and demonstrate that a relatively small electrostatic field can be used as a trigger for large deformations. We follow the theory of nonlinear electroelasticity [14,15], where the total stress is

$$\sigma_{ij}^{(t)} = \sigma_{ij}^{(c)} + \sigma_{ij}^{(m)}. \qquad (1)$$

Here, $\sigma_{ij}^{(c)}$ is the Cauchy *mechanical* stress, $\sigma_{ij}^{(m)} = \varepsilon\varepsilon_0 E_i E_j - (\varepsilon_0/2) E_n E_n \delta_{ij}$ is the Maxwell stress induced by the electric field E, ε is a dielectric modulus, and ε_0 is the vacuum permittivity. We assume that $\varepsilon = 6$, which is typical for common polymers, is constant [16]. We adopt a quite general constitutive law for *incompressible isotropic* materials, in which the principal stresses are

$$\sigma_k^{(c)} = \sum_{p=1}^{N} \mu_p \lambda_k^{\alpha_p} - q, \qquad (2)$$

where λ_k are the principal stretches, q is an arbitrary hydrostatic pressure, μ_p are shear moduli, and α_p are material constants. In the case $N=1$ with $\alpha_1 = 2$, the model (2), which is commonly denoted as Ogden model, reduces to the neo-Hookean one. With $N=3$ an excellent correlation with experimental data for elastomers is revealed [17]. Accordingly, we assume the following typical values for the elastic constants of soft polymers [17] $\mu_1 = 6.3 \times 10^5$ Pa, $\mu_2 = 1.2 \times 10^3$ Pa, $\mu_3 = -1 \times 10^4$ Pa, $\alpha_1 = 1.3$, $\alpha_2 = 5$, and $\alpha_3 = -2$. In the vicinity of the reference configuration this polymer behaves like a neo-Hookean material with shear modulus $\mu = \frac{1}{2}\sum_{p=1}^{3} \mu_p \alpha_p = 4.225 \times 10^5$ Pa.

Consider a spherical balloon made out of a dielectric elastomer with inner R_i and outer R_o radii. Here and thereafter, the notations $(\bullet)_i$ and $(\bullet)_o$ are used to specify quantities at the inner and outer radii, respectively. The thickness of the balloon wall is $H = R_o - R_i$. The inner and outer surfaces of the balloon wall are covered with thin electrodes with negligible elastic modulus [1]. With these electrodes electric field is induced across the wall. The balloon can be inflated with inner pressure P_i, and electrically exited with electric potential φ_o between the two electrodes. The associated boundary conditions are

$$\sigma_{rr}^{(t)}(r_i) = -P_i, \quad \sigma_{rr}^{(t)}(r_o) = 0, \quad \varphi(r_i) = 0, \quad \varphi(r_o) = \varphi_o, \qquad (3)$$

where r is the radius in the deformed configuration.

In spherical coordinate system the principal stretch ratios are

$$\lambda_r = \frac{dr}{dR} = \lambda^{-2}, \quad \lambda_\theta = \lambda_\phi = \frac{r}{R} = \lambda, \qquad (4)$$

where $\lambda(R) = (1 + (R_o/R)^3(\lambda_o^3 - 1))^{1/3}$. Maxwell equations reduce to Laplace equation for the electrical potential φ, which is solved in the deformed configuration. The components of the electric field $E \equiv -\nabla\varphi$ satisfying the electrostatic boundary conditions in (3) are

$$E_r = \frac{\varphi_o r_o r_i}{r_i - r_o}\frac{1}{r^2}, \quad E_\theta = E_\phi = 0. \qquad (5)$$

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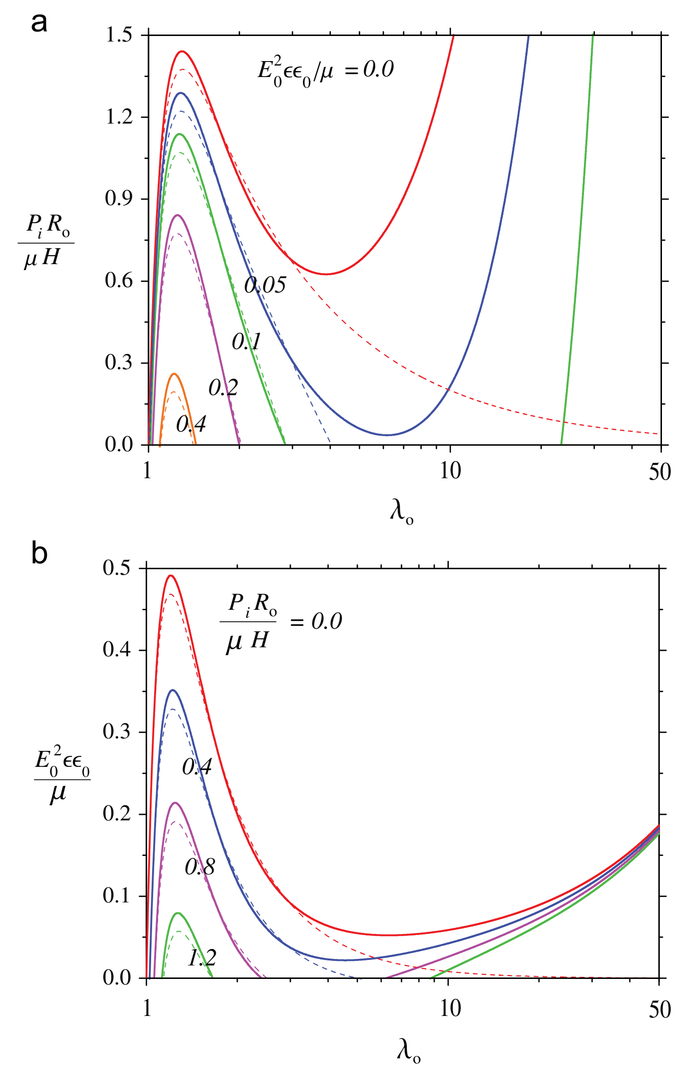

Fig. 1. Deformation of a thick-wall balloon due to (a) inflation pressure and (b) electrostatic excitation. The continuous and dashed curves correspond to Ogden and neo-Hookean materials, respectively.

The solution of the equilibrium equation

$$\frac{\mathrm{d}}{\mathrm{d}r}\sigma_{rr}^{(t)} = -\frac{2}{r}(\sigma_{rr}^{(t)} - \sigma_{\theta\theta}^{(t)}), \tag{6}$$

with boundary conditions (3) and relations (1), (2) and (4) reads

$$P_i = \sum_{p=1}^{N} \frac{2\mu_p}{3}\left\{ B\left[\lambda_i^{-3}, \lambda_o^{-3}, 1 - \frac{\alpha_p}{3}\right] + B\left[\lambda_o^{-3}, \lambda_i^{-3}, 1 + \frac{2\alpha_p}{3}\right]\right\}$$
$$+ \frac{1}{2t^2}\frac{(t-1)^2}{\lambda_i^2\lambda_o^2}\frac{t^4\lambda_i^4 - \lambda_o^4}{(t\lambda_i - \lambda_o)^2}\varepsilon\varepsilon_0 E_0^2, \tag{7}$$

where $B[z_0, z_1, a] = \int_{z_0}^{z_1} x^{a-1}(1-x)^{-1}\,\mathrm{d}x$ is the generalized incomplete Beta function, $t = R_i/R_o$, and $E_0 = \varphi_o/H$ is a referential electric field.

Results for a thick-wall balloon ($t = 0.8$) are presented in Fig. 1. Shown in Fig. 1(a) is the inflation pressure versus the stretch ratio for a few fixed electric fields. Fig. 1(b) shows the electric field versus the stretch ratio for a few values of the inflation pressure. The results for the Ogden and the neo-Hookean materials are denoted by continuous and dashed curves, respectively.

With the increase in the inflation pressure the balloon slowly expands until a critical pressure is reached. Further increase in the pressure leads to a sudden jump in the size of the balloon to a new stable state. This phenomenon is commonly denoted "snap-through". Application of electric field reduces the critical pressure at which the balloon snaps. We observe that this effect cannot be recovered with the neo-Hookean material [18]. Similarly, electrostatic excitation which is applied in the undeformed configuration leads to the expansion of the balloon up to a critical electric field. Further increase in the electric potential results in a snap-through of the balloon. When applied to a pre-inflated balloon, the critical electric field at which the instability occurs decreases. We note that the slopes of the curves along the secondary branches (the post-instability) are markedly lower than the ones along the primary branches. This implies that once the balloon snaps to its new state, smaller variations in the electric field result in larger actuations.

For thin-wall spheres [3] Eq. (7) can be simplified to

$$P_i = \frac{2\,H}{R}\left(\sum_{p=1}^{N}\mu_p(\lambda^{\alpha_p - 3} - \lambda^{-2\alpha_p - 3}) - \varepsilon_0\varepsilon E_0^2\lambda\right). \tag{8}$$

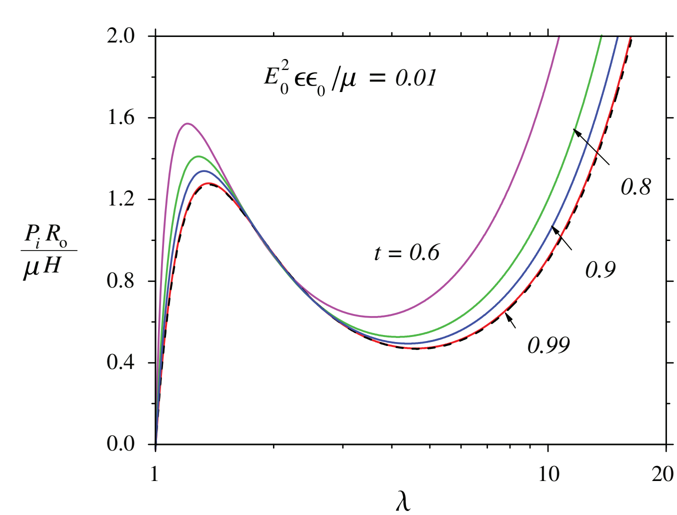

Fig. 2. The deformations of balloons with different wall thicknesses as functions of the pressure with fixed electric excitation. The dashed curve corresponds to the thin-wall approximation (8).

With $N=1$ and $\alpha_1=2$, Eq. (8) further reduces to the case of a thin-wall neo-Hookean sphere [18]. In Fig. 2(a) comparison of solution (7) with approximation (8) is carried out for different thickness ratios. We note that Eq. (8) provides a fair estimate for $t>0.9$. However, for micro-actuators with a diameter of a few tenth of micrometers this approximation will lead to considerable errors.

We consider next two snap-through cycles in which the inner pressure is fixed and the actuation is electrically controlled. To this end we show in Fig. 3 the deformation of a thin-wall balloon ($t=0.99$) as a function of (a) the inflation pressure and (b) the electrostatic excitation.

The first actuation cycle is "irreversible", and once the balloon deforms, it remains in a deformed state even after the removal of the electric field. This cycle is represented by the path A–B–C–D. Initially, the balloon is inflated, with $\varphi_o=0$, to point A. Application of a relatively low electric field will result in a expansion to point B, where the balloon snaps to a stable state at point C. Upon removal of the electrostatic field, the balloon shrinks to point D that corresponds to a stable configuration under zero electric excitation.

The second actuation cycle is "reversible" in the sense that upon removal of the electric excitation the balloon returns to the original configuration. This cycle corresponds to the path M–N–Q–S. As before, the balloon is initially inflated with $\varphi_o=0$, to point M. Application of electric field results in gradual inflation to point N, followed by abrupt expansion to point Q. Removal of the electric potential will result in shrinking of the balloon to point S, followed by a jump back to point M.

Lastly, in Fig. 4 we examine the usage of the dielectric balloon as a micro-pump. Consider a balloon with inlet and outlet unidirectional valves. When the balloon expands at a pressure lower than a specific negative threshold pressure P_1 the inlet valve opens and let liquid flow into the balloon. When the balloon shrinks, at some positive threshold pressure P_2 the outlet valve opens and liquid flows out. In a way of an example, in Fig. 4 the normalized pressures are $P_1=-0.29$ and $P_2=0.4$. The pumping cycle starts at point A at which the internal pressure is slightly lower than P_2. Once the balloon is excited, due to the electrostatic forces that act to shrink its wall, the internal pressure drops to P_1 at constant λ (since the balloon cannot deform as long as both valves are close). At this point the inlet valve opens, and the balloon expands while liquid flows in (segment B–C). At point C, due to the instability, the balloon further expands to point D. This is a stable point at which the inlet valve closes, terminating the sucking stage. As the electric excitation is removed, due to the elasticity of the balloon the inner pressure increases as long as the pressure is lower than P_2 (segment D–E). At point E the pressure reaches P_2, the outlet valve opens and liquid flows out of the balloon as it starts to shrink. This process continues till the balloon returns back to point A through point F.

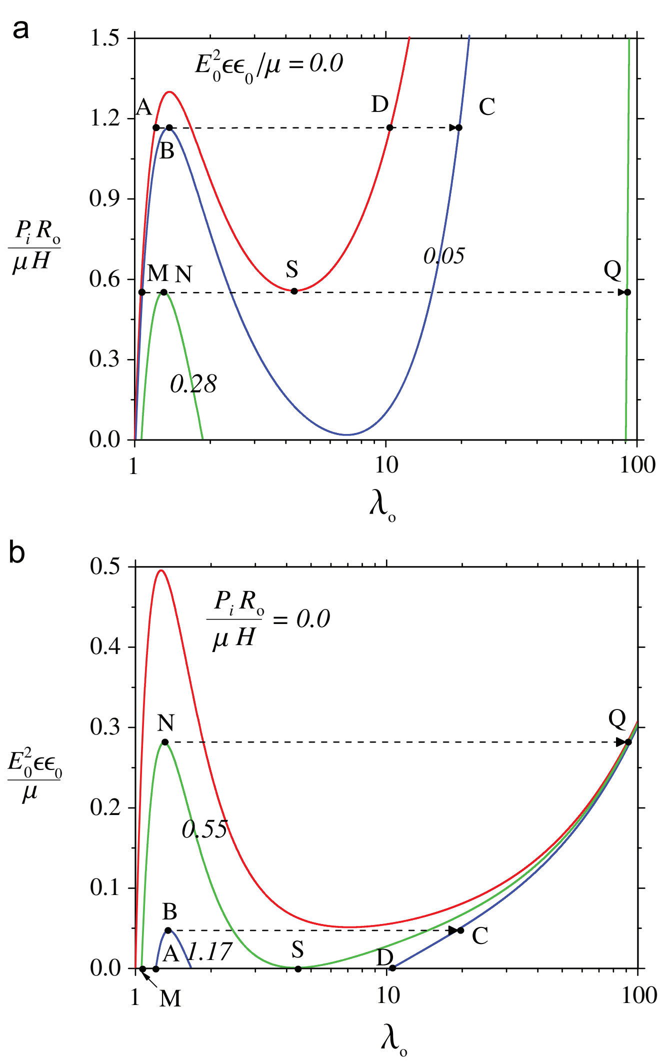

Fig. 3. Actuation cycles of a dielectric balloon subjected to (a) inflation pressure and (b) electrostatic excitation.

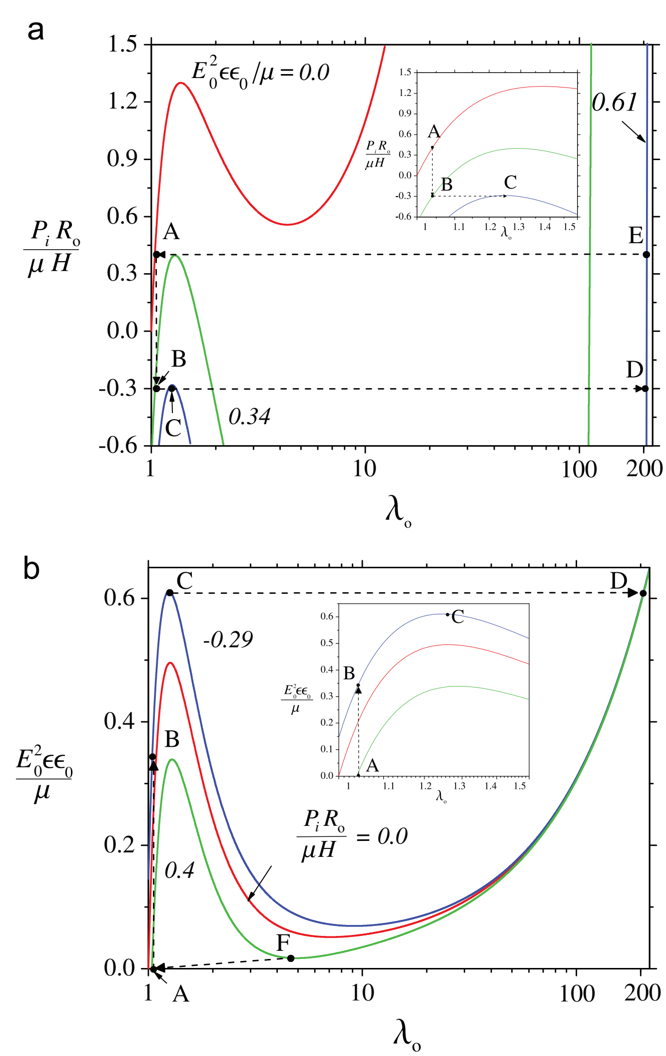

Fig. 4. Pumping cycle of a dielectric balloon subjected to (a) inflation pressure and (b) electrostatic excitation.

Acknowledgments

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